Devotions for the Skeptic

Part I. Mathematical Meditations

Introduction

In the same sense that few people come to faith in God through an intellectual process, few will come to faith through a mathematical process, for faith in God is neither intellectual nor mathematical but spiritual.

What do I mean by that?

If I claimed to have irrefutable proof that God exists what would be your inner thoughts? Are you excited about the prospect? Are you genuinely skeptical? Or are you ready to marshal every thought and argument to demolish any proof forthcoming? In the first two cases, you may be persuaded by the arguments but in the latter case it really doesn't matter what anyone says, does or demonstrates for, regardless of what evidence is put forward, you have already decided that it's false. It's not that you can't believe (you can) it's that you won't believe—period. And in that case, no mathematical proof in the world will convince you until you change from the inside. But for you that are in the spirit to at least entertain the possibility that God exists, read on.

Theologians have always understood that our revelations of God come through one of two experiential conduits: pain and awe. For those who are not facile with the language of math (described by Galileo as the language of God), your conduit will be pain, for the mathematical discoveries I intend to outline in this essay are exclusively awe inspiring for those that comprehend them and painful for those relegated to the ranks of the mathematically illiterate.

Mathematicians have also recognized the divine in their own discoveries. In the secular book *Is God a Mathematician?* author Mario Livio documents the debate as to whether math is an invention or whether it is a discovery and puts forth arguments for both. Perhaps as an occupational hazard, many of history's greatest mathematicians were people of deep religious or philosophical convictions including Leonhard Euler, Bernard Riemann and Gottfried Leibnitz. You may be surprised to learn that Isaac Newton wrote more about scripture than physics, optics, calculus, gravitation or any other topic for which he is renowned. The point is this: mathematical discoveries can have metaphysical as well as physical implications—it's part of the territory.

An interesting aspect of mathematics is that it is the domain of all nations throughout time. The language and discoveries of math have advanced through the ancient Greeks, Babylonians, Arabs, Italians, Hindi, English, Scottish, French, Germans, Swiss and many others as if authored by something transcendent to time, location and culture. Because of this, the language and expression of mathematics—*the universal language*-- is inherently void of bias and why I think a mathematical proof for the existence of God has the highest chance of any to convince the skeptical mind.

So let’s get on with it…

The Pythagoreans

Pythagoreanism was as much a religious vs. mathematical order of the classical world established by none other than Pythagoras of Samos. This should say something alone—the ancient Greeks who brought us geometry, classical education, philosophy, democracy, music, literature, medicine, drama and countless other bodies of knowledge that we still reference today could see the divine in the mathematical and vice versa. The two were synonymous.

Among their views was held a certain regard for integer numbers ($\mathbb{Z}$) and their ratios. Today we call such ratios *rational numbers* ($\mathbb{Q}$) but understand that our current concept of the number systems was quite different from the Pythagoreans: to them *all* numbers were rational:

$$x \in \mathbb{Q}\ if\ \exists\ m, n \in \mathbb{Z} : x = \frac{m}{n}\ , n \neq 0$$

Or in English:

”x is a rational number if there exists two integers m and n such that the quotient of m and n is equivalent to x and n can’t be zero because, well, just don’t go there”

Numbers like $\frac{1}{2}, \frac{2}{3}, 4\ \left(=\frac{4}{1}\right), \frac{-3}{8}, \frac{22}{7}$ or even outlandish fractional expressions like $\frac{123}{45777}\ or\ \frac{46784542}{958394583}$ are all rational numbers. Go crazy—for the combinations are limitless and no matter which two fractions you may generate that are infinitesimally close together, another rational number can be discovered that squeezes between. In fact:

Meditation #1

No matter which two rational numbers you concoct, you can also concoct an *infinite* number of other rational numbers that quantitatively fit between them according to the numerical “pecking order”.

For example, pick $\frac{1}{1000000}\ and\ \frac{2}{1000000}$. As small as they are, we can generate numbers in between:

$$\frac{1}{1000000} = \frac{10}{10000000} < \frac{11}{10000000} < \frac{12}{10000000} \ldots < \frac{19}{10000000} < \frac{20}{10000000} = \frac{2}{1000000}$$

Here I simply increased the denominator and numerator of each number by another order and generated a sequential set of numerators between the two. This sort of technique can be applied to two of the generated numbers, and so forth, *ad infinitum*.

The fact that rational numbers could be measured was somewhat central to Greek thinking too. In our very abstract world of today the addition of two numbers has no context: we are taught the mechanics of base 10 arithmetic at an early age and the operation of addition quickly becomes a rote recipe ultimately committed to a computer that does it with greater speed and as much understanding.

But when the ancient Greeks added two numbers it was the concatenation of two distances to create a third:

1 daktylos + 1 daktylos = 2 daktyloi

They didn't just add numbers for the hell of it—there was always a physical significance to it like distance, area, arc or the ratio of vibrating strings from which all Western music theory has rested for millennia. Needless to say it was important that these measurements (or numbers) could be distilled *exactly* from fundamentals.

Recall that Euclidean geometry taught us to find the midpoint of any length or split an angle perfectly in half (both being rational ratios of ½) with only the aid of a straight edge and compass for the simple reason that the first "master" rulers and protractors had to be developed first before it could be ripped off and massed produced by the Chinese.

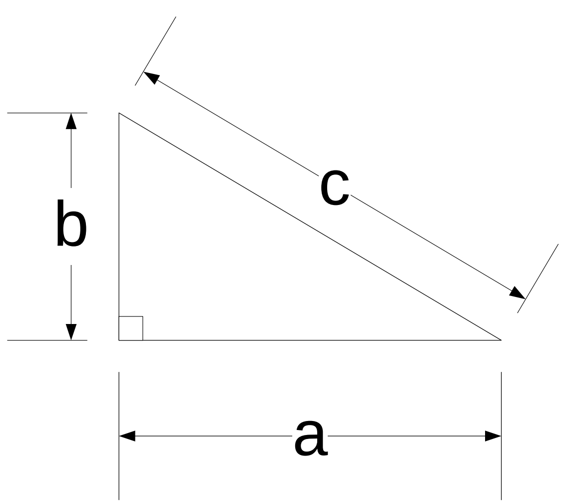
Meditation #2

Curiously, the Greeks were never successful at *trisecting* angles (a rational ratio of $\frac{1}{3}$). This along with other classical geometric problems was proven to be unsolvable by compass and straight edge construction, but only as late as the 19th century. Consider that what often appear as problems solvable through effort, ingenuity and the scientific progress, are indeed, unsolvable. And this is not the only one like it.

Now let us project Greek thinking to a property of the right triangle. If you are not acquainted with the famous Pythagorean Theorem it goes something like this:

$$a^2 + b^2 = c^2$$

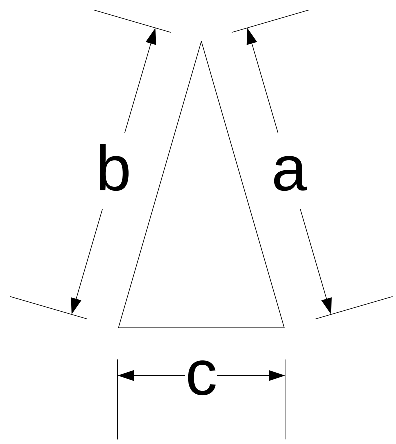
a property that holds for every right triangle so labeled:



In English, “the area of the square whose side is the hypotenuse (c, the side opposite the right angle) is equal to the sum of the areas of the squares whose sides are the two legs (a and b, the two sides that meet at a right angle).

Meditation #3:

In the classic Wizard of Oz movie, the scarecrow recites what we think is the Pythagorean Theorem as he is handed his diploma: "The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side." Mathematically:

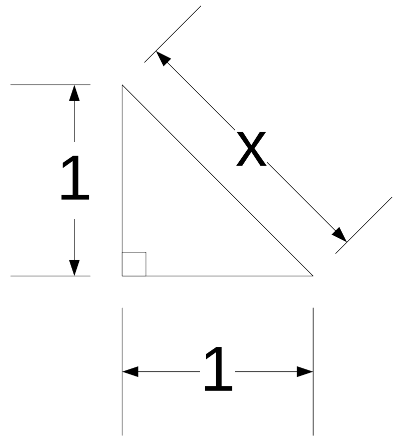


$$\sqrt{a} + \sqrt{b} \neq \sqrt{c}$$

Totally wrong—and easily dismissed by selecting a = b = 4 and c = 1. Dorothy should have slapped him; Toto should have mauled him. Obviously, his diploma was not in mathematics. In any event, even the most high-sounding intellectual with a diploma can be outed when it comes to the language of math. In order for a mathematical theorem to be totally debunked one merely needs to scare up a single counter example; on the other hand, in order to be true, it has to be rigidly demonstrated true for all cases (without actually cycle through all the cases which would take in excess of forever even with a massive computer).

By the way, unlike a theorem, a *conjecture* is a statement a mathematician thinks is altogether true but is too lazy to prove it. As long as no one comes up with an example that renders the statement false, the idea can dwell in the netherworld of conjecture conferring honor to the inventor in perpetuity. Nice.

So then, between the seasons of war among city-states and the odd Median invader, the Greeks fiddled with math and took, as a case study, the standard case for the Pythagorean Theorem:



Seems like a reasonable basis to use a = b = 1 since such experiments might provide generalized solutions of scale as we might observe in trigonometry when radius = 1. At any rate the task was to find *x* using the Pythagorean Theorem which should be a rather trivial exercise:

$$1^2 + 1^2 = x^2$$

$$2 = x^2$$

$$\mathrm{x} = \sqrt{2}$$

Except that the notation or concept of $\sqrt{2}$ did not exist as we know it today; what the Greeks were ***really*** looking for was:

$$\frac{\mathrm{m}}{\mathrm{n}} = \sqrt{2}$$

Careful to point out that $\mathrm{n} \neq 0$. Remember now that in the Greek mind there was supposed to be a solution representative in this manner, i.e., one should be able to find a small enough measurement by which a countable number of them will span our hypotenuse. Seems easy—right? So then, get it done and make is snappy before the Spartans get bored!

Well, legend has it that the Pythagorean, Hippasus of Metapontum, investigating this problem stumbled on what are called irrational numbers and was drowned at sea as a mathematical heretic. And that was only the tip of it.

Meditation #4

There is a simple proof-by-contradiction that shows there is no such pair m and n. Because of this, the square root of two is not a rational number ($x \notin \mathbb{Q}$) by definition. It is irrational (sorry, no fancy bold face letter representative of the set of irrational numbers—an obvious snub dating back to the Pythagoreans) and a member of the real numbers ($\mathbb{R}$) which includes rational and irrational numbers together in one bracketed fenced-in area and heavily guarded. We know this now.

But let’s not diminish the awe this must have initially inspired: given the *infinity* of opportunities and selections, there is NO rational representation for the square root of two. Infinity is a large number—correction—it’s even larger. Surely there must be one combination out there that can put us smack dab in the middle of square root of two. Remember that we can “go crazy” but, alas, no amount of crazy will solve this one without actually going crazy.

Meditation #5

Once upon a time, no one ever heard of irrational numbers—they were as rare as hen’s teeth. But a branch of mathematics called Real Analysis subjects the unwary student to the *proof* that irrational numbers actually *outnumber* rational ones by a 1:inifinity ratio. This too is baffling since it was demonstrated earlier that I can concoct an infinite number of rational numbers between any two rational numbers you select. NOW they tell us that there’s an infinite number of these irrational vermin swarming in between the infinite rational numbers concocted. I wonder what’s swarming between the irrational ones? And on this order, what does “between” exactly mean? Does your head hurt?

Divine summary #1

Irrational numbers are a mathematical insight to the existence and character of God:

- Given an infinite number of chances something WON’T necessarily happen—particularly the universe and your existence.
- God is between and among anything and everything you can conceive of let alone quantify, and He pretty much outnumbers you infinitely.
- God may not be apparent to your rational sensibilities and yet, is everywhere waiting for you to discover Him. Imagine the awe (or the pain) when you do.